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The first result is the same as the average distance of a point in a circle from a point in its circumference. The second result is the same as the average distance between two points on the circumference of a circle.

Also solved by *F. P. MATZ*. Professor Matz in his solution did not go through the details of integration as Professor Zerr has done.

II. Solution by *L. C. WALKER, A. M.*, Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Let P, Q be the random points, O the center of the hemisphere, and A the vertex. Put $OP=r$, $\angle POQ=\theta$, $\angle AOP=\phi$, M_1 =average length of the straight line, and M_2 =average length of the arc of a circle, which joins the points.

Now while θ is constant, and $<\frac{1}{2}\pi$, and $\phi<\frac{1}{2}\pi-\theta$, for each position of P, Q may be taken anywhere in the circumference of a small circle whose pole is P , and radius $r\sin\theta$.

But when $\theta<\frac{1}{2}\pi$, and $\phi>\frac{1}{2}\pi-\theta$, or when $\theta>\frac{1}{2}\pi$, and $\phi>\theta-\frac{1}{2}\pi$, for each position of P, Q may be taken anywhere in the arc of a small circle whose pole is P , and length $2r\sin\theta[\pi-\cos^{-1}(\cot\theta\cot\phi)]$.

Hence, if θ is of given value, and $<\frac{1}{2}\pi$, the number of ways the two points can be taken is

$$\int_0^{\frac{1}{2}\pi-\theta} 2\pi r \sin\theta \cdot 2\pi r \sin\phi \cdot r d\phi + \int_{\frac{1}{2}\pi-\theta}^{\frac{1}{2}\pi} 2r \sin\theta [\pi - \cos^{-1}(\cot\theta\cot\phi)] \cdot 2\pi r \sin\phi \cdot r d\phi \\ = 4\pi r^3 (\pi - \theta) \sin\theta.$$

If $\theta>\frac{1}{2}\pi$, the number of ways the two points can be taken is

$$\int_{\theta-\frac{1}{2}\pi}^{\frac{1}{2}\pi} 2r \sin\theta [\pi - \cos^{-1}(\cot\theta\cot\phi)] \cdot 2\pi r \sin\phi \cdot r d\phi = 4\pi r^3 (\pi - \theta) \sin\theta.$$

Hence, since the whole number of ways the two points can be taken is $4\pi^2 r^4$, we have

$$M_1 = \frac{1}{4\pi^2 r^4} \int_0^\pi 4\pi r^3 (\pi - \theta) \sin\theta \cdot 2r \sin\frac{1}{2}\theta \cdot r d\theta = \frac{32r}{9\pi}, \text{ and}$$

$$M_2 = \frac{1}{4\pi^2 r^4} \int_0^\pi 4\pi r^3 (\pi - \theta) \sin\theta \cdot r\theta \cdot r d\theta = \frac{4r}{\pi}.$$

MISCELLANEOUS.

101. Proposed by *G. B. M. ZERR, A. M.*, Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

A wire is laid along the surface of a right cone semi-vertical angle β so that it cuts the generators everywhere at a constant angle θ . Find the radius of curvature and radius of torsion.

Solution by the PROPOSER.

Let s = length of wire from origin to any point, ρ = radius of curvature, $1/\sigma$ = radius of torsion. From problem 85, Calculus, No. 4, Vol. VI., we get $x =$

$$r \cos \varphi, y = r \sin \varphi, z = s \cos \beta \cos \theta, \varphi = \frac{\tan \theta}{\sin \beta} \log s, r = s \sin \beta \cos \theta, dx = \cos \varphi dr - r \sin \varphi d\varphi,$$

$$dy = \sin \varphi dr + r \cos \varphi d\varphi, dz = \cos \beta \cos \theta ds, d\varphi = \frac{\tan \theta}{\sin \beta} \cdot \frac{ds}{s}, dr = \sin \beta \cos \theta ds.$$

$$\therefore dx/ds = \sin \beta \cos \theta \cos \varphi - \sin \theta \sin \varphi, dy/ds = \sin \beta \cos \theta \sin \varphi + \sin \theta \cos \varphi,$$

$$dz/ds = \cos \beta \cos \theta, d^2x/ds^2 = -\frac{\sin \theta}{s} \left(\sin \varphi + \frac{\tan \theta \cos \varphi}{\sin \beta} \right),$$

$$d^2y/ds^2 = \frac{\sin \theta}{s} \left(\cos \varphi - \frac{\tan \theta \sin \varphi}{\sin \beta} \right), d^2z/ds^2 = d^3z/ds^3 = 0,$$

$$d^3x/ds^3 = -\frac{\sin \theta \tan \theta}{s^2 \sin \beta} \left(\cos \varphi - \frac{\tan \theta \sin \varphi}{\sin \beta} \right), d^3y/ds^3 = -\frac{\sin \theta \tan \theta}{s^2 \sin \beta} \left(\sin \varphi + \frac{\tan \theta \cos \varphi}{\sin \beta} \right)$$

$$1/\rho^2 = (d^2x/ds^2)^2 + (d^2y/ds^2)^2 + (d^2z/ds^2)^2 = \frac{\sin^2 \theta}{s^2} \left(1 + \frac{\tan^2 \theta}{\sin^2 \beta} \right).$$

$$\begin{aligned} \frac{1}{\rho^2 \sigma} &= \begin{vmatrix} \frac{dx}{ds}, & \frac{dy}{ds}, & \frac{dz}{ds} \\ \frac{d^2x}{ds^2}, & \frac{d^2y}{ds^2}, & \frac{d^2z}{ds^2} \\ \frac{d^3x}{ds^3}, & \frac{d^3y}{ds^3}, & \frac{d^3z}{ds^3} \end{vmatrix} = \frac{dz}{ds} \left(\frac{d^2x}{ds^2} \cdot \frac{d^3y}{ds^3} - \frac{d^3x}{ds^3} \cdot \frac{d^2y}{ds^2} \right) \\ &= \frac{\sin^2 \theta \tan \theta}{s^3 \sin \beta} \left(1 + \frac{\tan^2 \theta}{\sin^2 \beta} \right) \cos \beta \cos \theta. \end{aligned}$$

$$\therefore 1/\sigma = \frac{\tan \theta \cos \beta \cos \theta}{s \sin \beta} = \frac{\sin \theta \cos \beta}{s \sin \beta}$$

Also solved by WILLIAM HOOVER.

102. Proposed by CHARLES C. CROSS, Whaleyville, Va.

Required the least multiple of 17 which when divided by 2, 3, 4, . . . 16 leaves in each case 1 as a remainder.

I. Solution by MARVIN E. SMITHEY, A. M., Instructor in Mathematics, Randolph-Macon Academy, Bedford City, Va.

$x(2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13) + 1$, x being any positive integer, represents all numbers satisfying the given condition. In order to determine what value of x will make this number the least multiple of 17, the following equation must be solved in least positive integers.